

Econ 301: Managerial Economics

Topic 4: Elasticity

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The story so far

- So far, We have looked at ‘qualitative’ changes (direction)
 - If income increases does demand for a good increase or decrease?
 - If price of a substitute/complement increases demand increases/decreases.
 - If input prices increase, supply of a good decreases.
- However, we want to know more than just the directional change. We also want to know by how much (quantitative changes)!
 - If Income increases by 1% how much more demand of a normal good?
 - 0.5%?
 - 1%?
 - 50%?
- Elasticity helps us understand the magnitude of changes

What is Elasticity?

- A measure of the responsiveness of one variable to changes in another variable; the percentage change in one variable that arises due to a given percentage change in another variable
- The elasticity between two variables, G and S , is mathematically expressed as:

$$E_{G,S} = \frac{\% \Delta G}{\% \Delta S}$$

- When a functional relationship exists, like $G=f(S)$, the elasticity is

$$E_{G,S} = \frac{dG}{dS} \frac{S}{G}$$

Overview

$$G = f(S) , \text{ or } Q_x = f(P_x, P_y, M, H)$$

$$E_{G,S} = \frac{dG}{dS} \frac{S}{G}$$

- Own Price Elasticity: $E_{Q_x, P_x} = \frac{dQ_x}{dP_x} \frac{P_x}{Q_x}$
- Cross Price Elasticity: $E_{Q_x, P_y} = \frac{dQ_x}{dP_y} \frac{P_y}{Q_x}$
- Income Elasticity: $E_{Q_x, M} = \frac{dQ_x}{dM} \frac{M}{Q_x}$

How do we get that result?

- Recall one way of calculating percentage change

$$\% \text{change} = \frac{\text{Change}}{\text{Original}}$$

- For example, if new quantity is 12 and old quantity is 10,

$$\% \text{change} = \frac{12-10}{10} = \frac{2}{10} = 0.2 = 20\%$$

- So, % change in quantity demanded is $\% \text{change} = \frac{\text{Change}}{\text{Original}} = \frac{\Delta Q}{Q}$

- And % change in Price is $\% \text{change} = \frac{\text{Change}}{\text{Original}} = \frac{\Delta P}{P}$

Price elasticity of demand

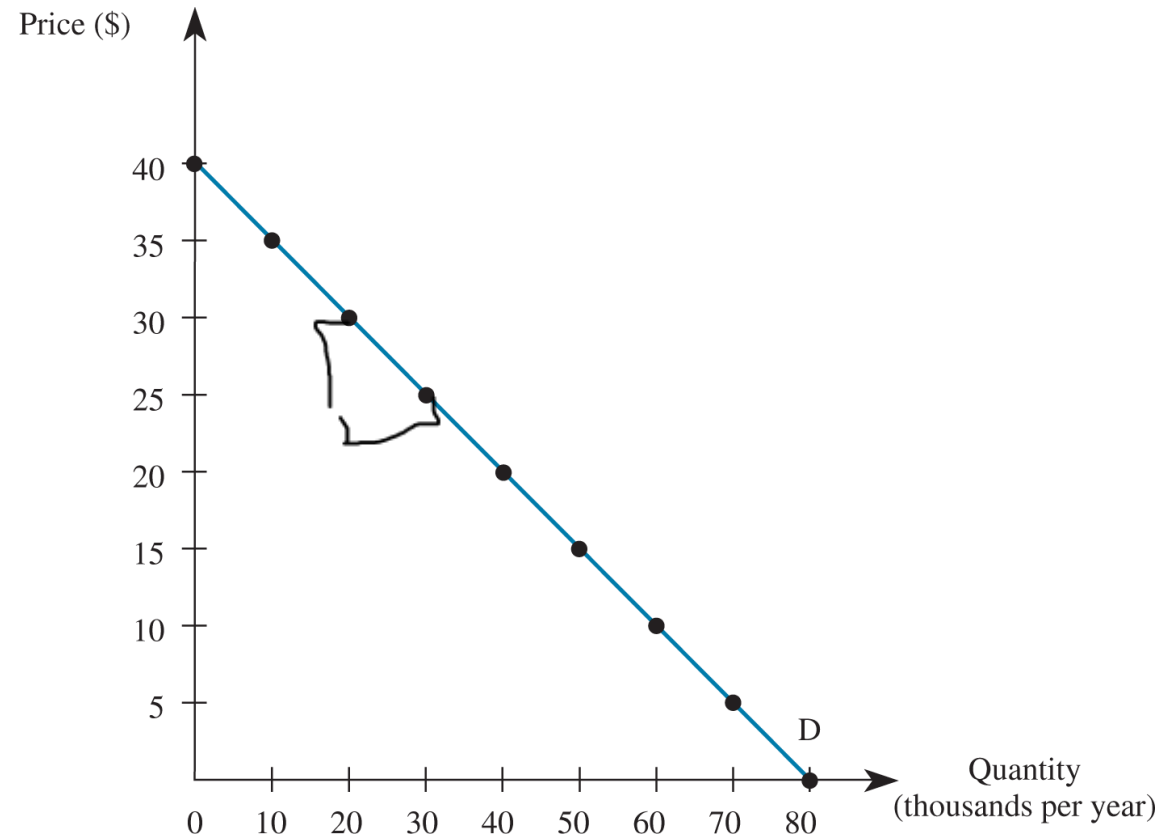
- Price Elasticity of Demand (ϵ^d) is thus

$$\frac{\% \Delta Q^d}{\% \Delta P} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\Delta Q P}{\Delta P Q} = \frac{\Delta Q}{\Delta P} * \frac{P}{Q}$$

- Note: $\frac{\Delta Q}{\Delta P}$ is just the inverse slope of the inverse demand function ($P=A+BQ$)....
 $\frac{1}{\text{slope}}$
- Why?

Price elasticity of demand

- Remember the slope is simply Rise/Run



Price elasticity of demand

- Slope of demand curve = $\frac{Rise}{Run} = \frac{\Delta P}{\Delta Q}$

- Inverse of slope of demand curve:

$$\frac{1}{slope} = \frac{1}{\frac{rise}{run}} = \frac{1}{\frac{\Delta P}{\Delta Q}} = \frac{\Delta Q}{\Delta P}$$

- To make it even easier! The slope of the demand curve Q is just the inverse of slope of P=

$$Q = A - PB$$

$$BP = A - Q$$

$$P = \frac{A}{B} - \frac{1}{B} * Q$$

- If $\Delta Q = 1$, then $\Delta P = -\frac{1}{B}$

Example #1

- $P = 100 - 2Q$
- Therefore, $Q = 50 - 0.5P$
- If price is \$10, what is ϵ^d ?

Example #2

- $P=100-2Q$
- Therefore, $Q=50-0.5P$
- At what price is the demand curve unit elastic($=-1$)?

Example #2

Measurement Aspects of Elasticity

- Important aspects of the elasticity:
- Sign of the relationship:
 - Positive
 - Negative
- Absolute value of elasticity magnitude relative to unity:
- $|E_{G,S}| > 1$ G is highly responsive to changes in S .
- $|E_{G,S}| < 1$ G is slightly responsive to changes in S .

Own Price Elasticity of Demand

- **Own price elasticity of demand**
- Measures the responsiveness of a percentage change in the quantity demanded of good X to a percentage change in its price

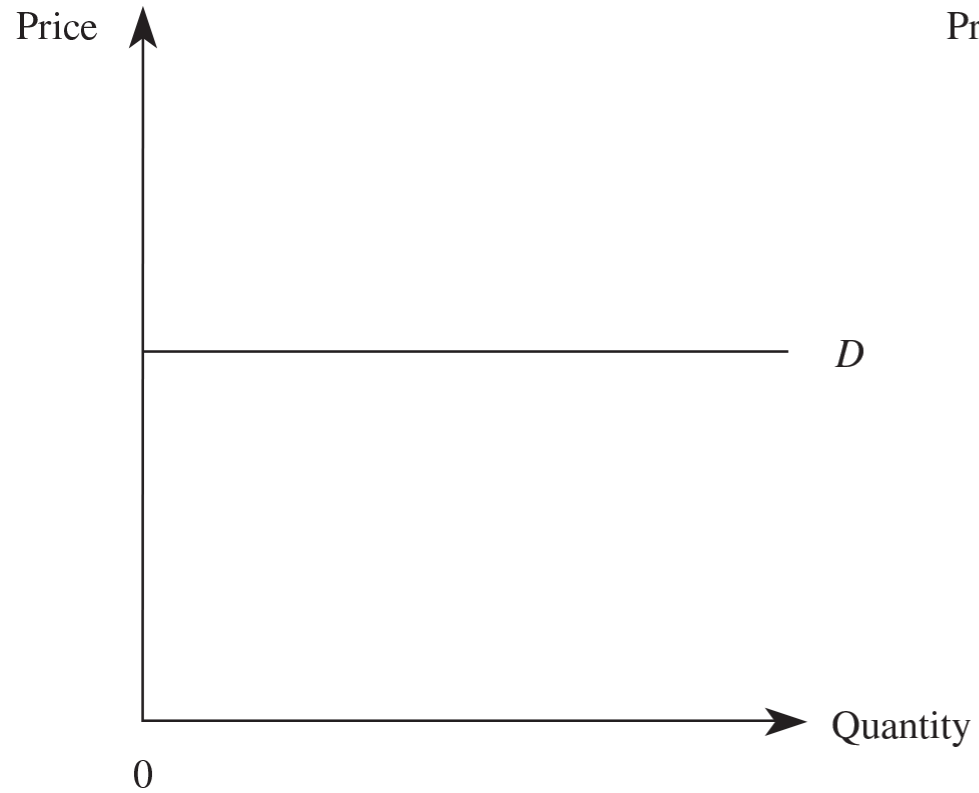
$$E_{Q_x, P_x} = \frac{\% \Delta Q_x^d}{\% \Delta P_x}$$

- Sign: negative by law of demand
- Magnitude of absolute value relative to unity:
- $|E_{Q_x, P_x}| > 1$: Elastic
- $|E_{Q_x, P_x}| < 1$: Inelastic
- $|E_{Q_x, P_x}| = 1$: Unitary elastic

What does this actually mean?

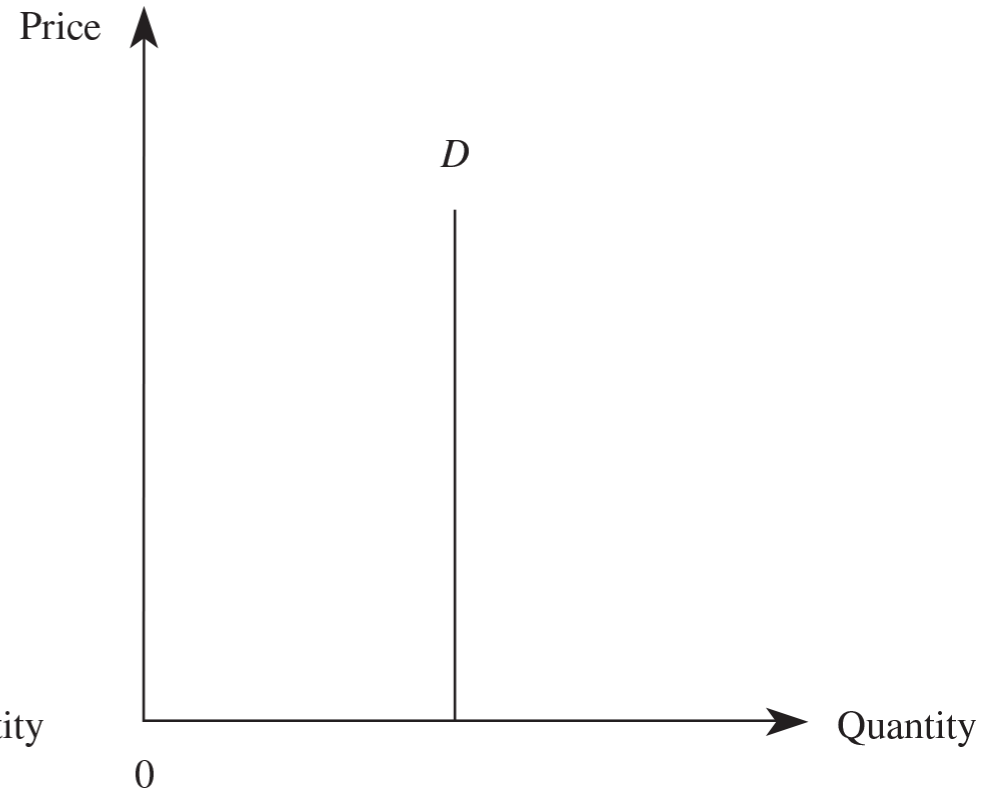
- Elasticity of demand measures the responsiveness of consumer desires for a product when price changes.
- Elastic means that people are more responsive to price changes of a product (demand increases (decreases) by $>1\%$ if prices decrease (increase) by 1%).
- Inelastic means that people are less responsive to price changes of a product (if prices rise/fall, people will still demand the same amount of the good).

Elasticity



Perfectly
Elastic
 $E_{Q_x, P_x} = -\infty$

(a)



Perfectly
Inelastic
 $E_{Q_x, P_x} = 0$

(b)

Factors Affecting the Own Price Elasticity

Three factors can impact the own price elasticity of demand:

1. Availability and number of substitutes
2. Time/duration of purchase horizon
3. Expenditure share of consumers' budgets

Elasticity on the Linear Demand Curve

- Is elasticity the same at every point on the linear demand curve?

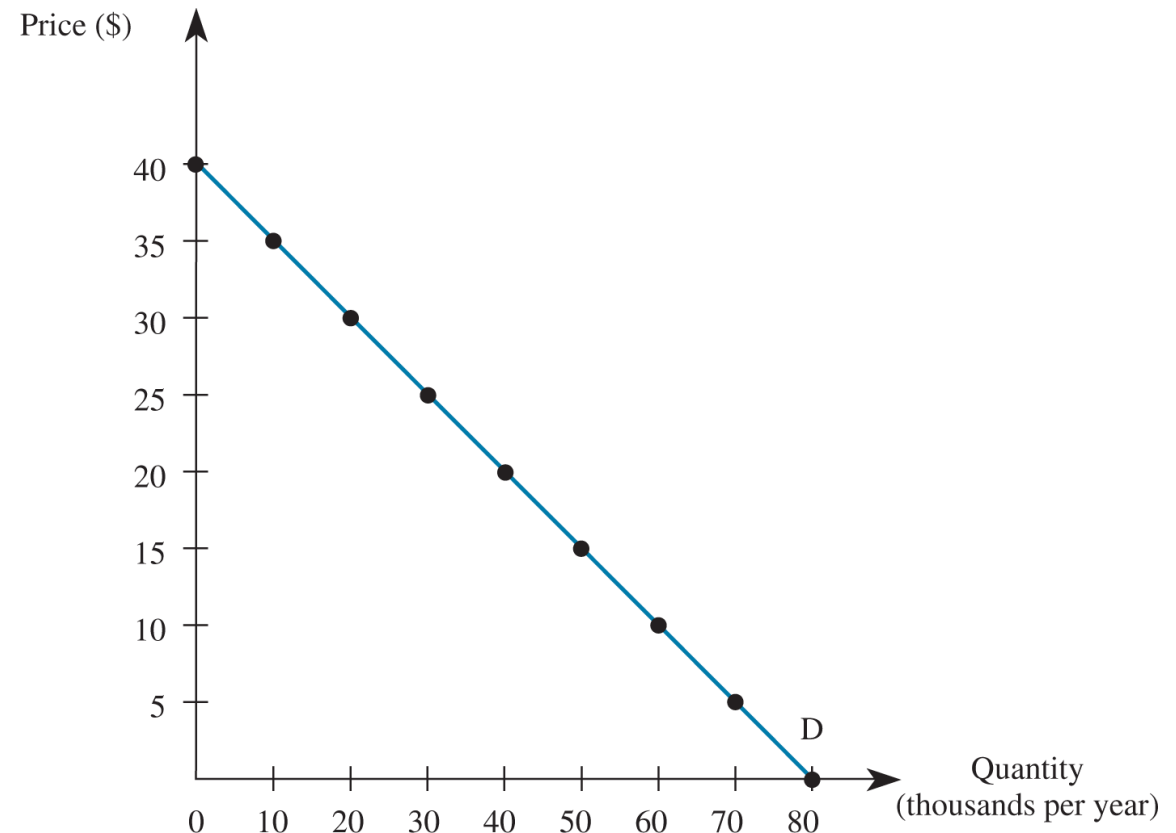
- No! This is a common mistake.

- Remember $\epsilon_{Q_x, P_x} = \frac{dQ_x}{dP_x} \frac{P_x}{Q_x}$

- The slope of the linear demand curve is just

$$\frac{1}{\frac{dQ_x}{dP_x}} = \frac{dP_x}{dQ_x}$$

- $\frac{P_x}{Q_x}$ is changing at each point!



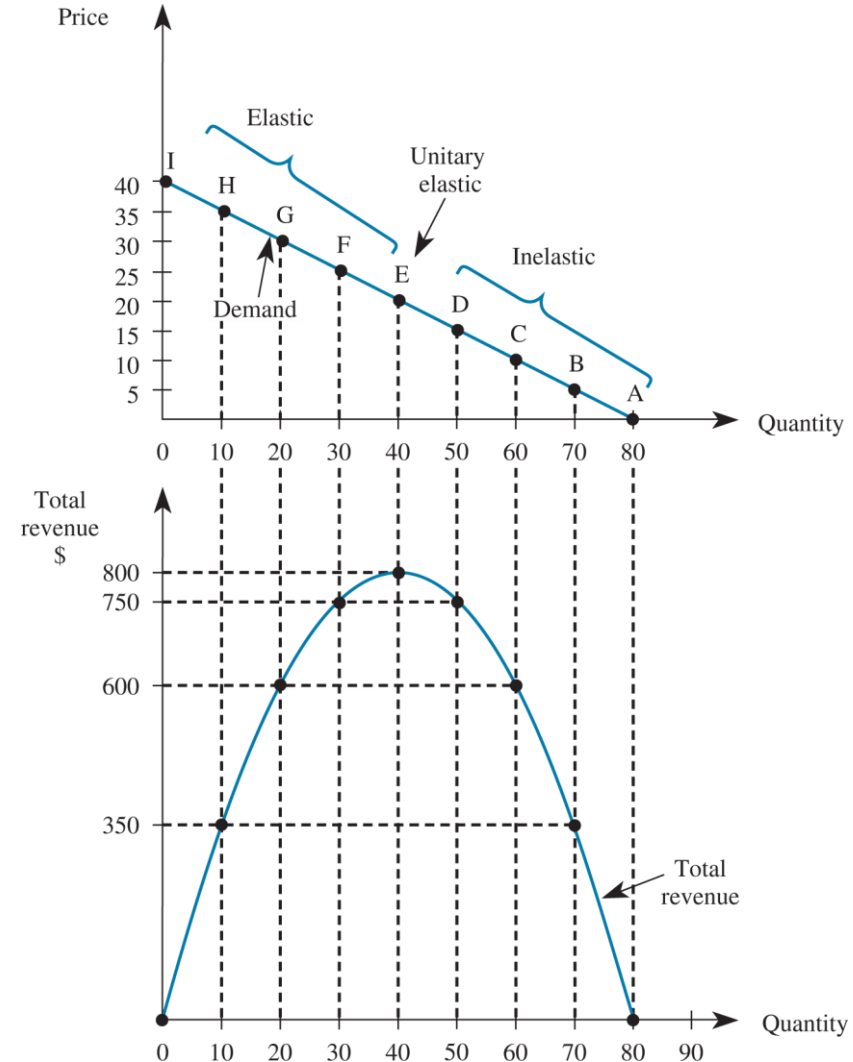
Total Revenue and Elasticity

Total Revenue ($Q \cdot P$) and Elasticity $Q_x^d = 80 - 2P_x$

	Price of Software (P_x)	Quantity of Software Sold (Q_x)	Own Price Elasticity (E_{Q_x, P_x})	Total Revenue ($P_x Q_x$)
A	\$ 0	80	0.00	\$ 0
B	5	70	-0.14	350
C	10	60	-0.33	600
D	15	50	-0.60	750
E	20	40	-1.00	800
F	25	30	-1.67	750
G	30	20	-3.00	600
H	35	10	-7.00	350
I	40	0	$-\infty$	0

Demand, Elasticity, and Total Revenue

- Total Revenue is maximized when elasticity = 1 (unitary elastic).
- In the elastic range, total revenue can be increased by decreasing price.
- In the inelastic range, total revenue can be increased by increasing price.



Total Revenue & Marginal Revenue

- Recall that Total Revenue=Price*Quantity

$$TR=P(Q)*Q \text{ or } TR=P*Q(P)$$

- Example: $P = 100 - 2Q$, then
- $TR = P(Q)*Q$
- $TR = (100 - 2Q)Q$
- $TR = 100Q - 2Q^2$

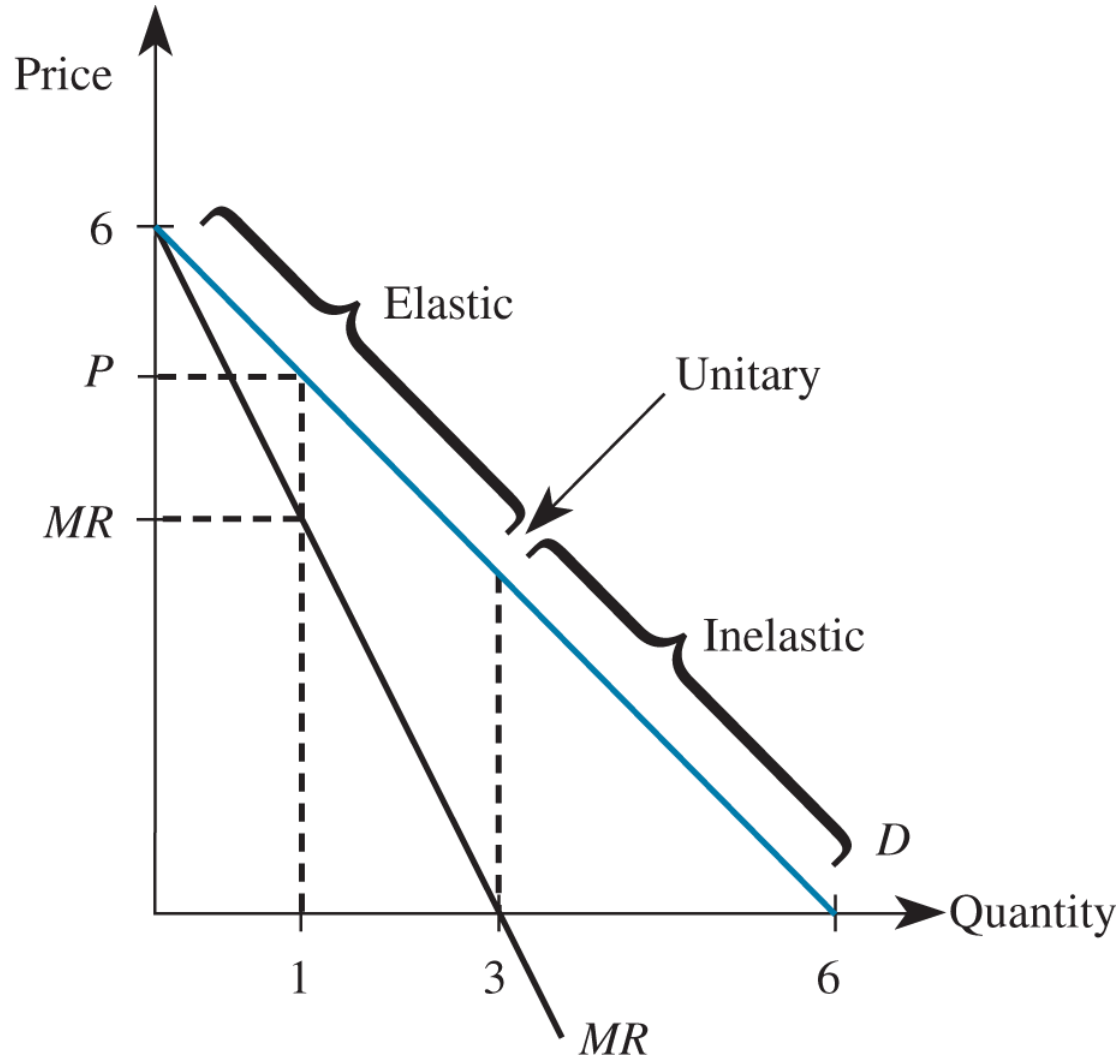
- Marginal Revenue= $\frac{\Delta TR}{\Delta Q}$

This is just $\frac{dTR}{dQ} = 100 - 4Q$

Total Revenue & Marginal Revenue

- When demand is elastic, the percentage change in quantity is larger than the percentage change in price -> lowering prices will increase quantity demanded **more** than the decrease in price -> total revenue is increasing -> Marginal revenue is POSITIVE (>0) when demand is elastic.
- When demand is inelastic, the percentage change in quantity is smaller than the percentage change in price -> lowering prices will increase quantity demanded **less** than the decrease in price -> total revenue is decreasing -> Marginal revenue is NEGATIVE (<0) when demand is inelastic.
- When demand is unit elastic, the percentage change in quantity is equal to the percentage change in price -> lowering prices will increase quantity demanded by the **same** amount as the decrease in price -> total revenue is not changing (has maximized) -> Marginal revenue is EQUAL TO ZERO ($=0$) when demand is unit elastic

Total Revenue & Marginal Revenue



• [Access the text alternative for slide images.](#)

- Why is MR lower than demand?
 - Price isn't just lowered on the next good but all goods!
- At \$5, 1 unit is bought (MR is \$5).
- To entice more purchases sellers need to reduce price to \$4
- At \$4, 2 units bought. TR= \$8, MR=\$3
 - $MR > 0$, and $MR < \text{Price}$
- At \$3, 3 units bought. TR= \$9, MR=\$1
 - $MR > 0$, and $MR < \text{Price}$

Marginal Revenue and the Own Price Elasticity of Demand

- The *marginal revenue* can be derived from a market demand curve.
 - Marginal revenue measures the additional revenue due to a change in output.
- This link relates *marginal revenue* to the own price elasticity of demand as follows: Where
 - E =Own Price elasticity of demand
 - P =Price

$$MR = P \left[\frac{1 + E}{E} \right]$$

- When $-\infty < E < -1$ then, $MR > 0$.
- When $E = -1$ then, $MR = 0$.
- When $-1 < E < 0$ then, $MR < 0$.

Total Revenue & Marginal Revenue

Example #1: $P=100-2Q$, $P=\$10$ and $E=-0.11$ (we know this is inelastic so $MR<0$).

Example #2: $P=100-2Q$, $P=\$70$ and $E=-2.33$ (we know this is elastic so $MR>0$).

Cross-Price Elasticity

- **Cross-price elasticity**
- Measures the responsiveness of a percent change in demand for good X due to a percent change in the price of good Y

$$E_{Q_x, P_y} = \frac{\% \Delta Q_x^d}{\% \Delta P_y}$$

- If $E_{Q_x, P_y} > 0$, then X and Y are substitutes.
- If $E_{Q_x, P_y} < 0$, then X and Y are complements.

Cross-Price Elasticity

Example: Let demand be represented by $Q^d = -2P_x - 0.5P_y + 3M$

- Where: $M = \$10$, $P_y = \$4$, $P_x = \$12 \rightarrow Q^d = -24 - 2 + 30 = 4$

$$\epsilon_{Q_x, P_y} = \frac{dQ_x}{dP_y} \frac{P_y}{Q_x} =$$

- These two goods are complements in consumption.

Cross-Price Elasticity in Action

- Suppose it is estimated that the cross-price elasticity of demand between clothing and food is -0.18 . If the price of food is projected to increase by 10 percent, by how much will demand for clothing change?

$$-0.18 = \frac{\% \Delta Q_{Clothing}^d}{10} \Rightarrow \% \Delta Q_{Clothing}^d = -1.8$$

- That is, demand for clothing is expected to decline by 1.8 percent when the price of food increases 10 percent.

Another Thing

- Suppose your revenue is based on more than one good, and those goods are related.

Example: You own a food truck that sells burgers and fries

- If you change the price of fries, it will affect the revenue you get from both fries and its related good, burgers
- So, your revenue change is going to depend on the own price elasticity of demand for fries, but also the cross-price elasticity of demand for burgers and fries
- Note that this would work for substitutes as well, like fries and onion rings.

Cross-Price Elasticity Revenue

- Assessing the overall change in revenue from a price change for one good when a firm sells two goods is:

$$\Delta R = \left[R_x \left(1 + E_{Q_x, P_x} \right) + R_y E_{Q_y, P_x} \right] \times \% \Delta P_x$$

Let's Break This Down

$$\Delta R = \left[R_x \left(1 + E_{Q_x, P_x} \right) + R_y E_{Q_y, P_x} \right] \times \% \Delta P_x$$

- $\% \Delta P_x$ how much you are changing the price of fries.
- ΔR is the change in total revenue. This will depend on:
 - R_x the revenue you get from fries
 - R_y the revenue you get from Burgers
- But how much the revenue you get from fries is going to depend on the price elasticity of demand for fries

$$R_x(1 + \epsilon_{Q_x^d, P_x})$$

- And how much revenue you get from burgers is going to depend on the cross price elasticity

$$R_y(1 + \epsilon_{Q_y^d, P_x})$$

Example

- A food truck knows that
- the price elasticity of demand for fries is -0.8
- The cross-price elasticity of demand for fries and burgers is -2.
- They currently sell 100 fries at \$1 each and 40 burgers at \$2 each.
- How much will revenues change if they increase the price of fries 25%?

$$\Delta R = \left[R_x \left(1 + E_{Q_x, P_x} \right) + R_y E_{Q_y, P_x} \right] \times \% \Delta P_x$$

Example

$$\Delta R = [R_x(1 + \epsilon_{Q_x^d, P_x}) + R_y(\epsilon_{Q_y^d, P_x})] * \% \Delta P_x$$

$$\Delta R = [R_x(1 + \epsilon_{Q_x^d, P_x}) + R_y(\epsilon_{Q_y^d, P_x})] * 25\%$$

$$\Delta R = [(100 * \$1)(1 + \epsilon_{Q_x^d, P_x}) + (2 * \$40)(\epsilon_{Q_y^d, P_x})] * 25\%$$

$$\Delta R = [(100 * \$1)(1 - 0.8) + (2 * \$40)(-2)] * 25\%$$

$$\Delta R = [(100 * 0.2) + (\$80 * -2)] * 25\%$$

$$\Delta R = [(\$20 - \$160) * 0.25] = -\$35$$

Income Elasticity

- **Income elasticity**

- Measures the responsiveness of a percent change in demand for good X due to a percent change in income

$$E_{Q_x, M} = \frac{\% \Delta Q_x^d}{\% \Delta M}$$

- If $E_{Q_x, M} > 0$, then X is a ***normal good***.
- If $E_{Q_x, M} < 0$, then X is an ***inferior good***.

Income Elasticity

Example: Let demand be represented by $Q^d = -2P_x - 0.5P_y + 3M$

- Where: $M = \$10$, $P_y = \$4$, $P_x = \$12 \rightarrow Q^d = -24 - 2 + 30 = 4$

$$\epsilon_{Q_x, M} = \frac{dQ_x}{dM} \frac{M}{Q_x} =$$

- That means this is a normal good as $\epsilon_{Q_x, M} > 0$

Income Elasticity in Action

- Suppose that the income elasticity of demand for organic potatoes is estimated to be 2.26. If income is projected to decrease by 10 percent, what is the impact on the demand for organic potatoes?

$$2.26 = \frac{\% \Delta Q_x^d}{-10}$$

- Demand for organic potatoes will decline by 22.6 percent.
- Are organic potatoes a normal or inferior good?
- Since demand decreases as income declines, organic potatoes are a normal good.

Elasticities in general

- If you can measure it, you can calculate an elasticity!

Examples:

- How responsive is the quantity demanded for your good to the distance people have to drive to get it?
- How responsive is the quantity demanded of your good to wait times?
- How responsive is the quantity demanded of your good to advertising?

Other Elasticities

- ***Own advertising elasticity*** of demand for good X is the ratio of the percentage change in the consumption of X to the percentage change in advertising spent on X .
- ***Cross-advertising elasticity*** between goods X and Y would measure the percentage change in the consumption of X that results from a 1 percent change in advertising toward Y .

Example: Elasticity of Advertising

$$\epsilon_{Q,A} = \frac{dQ}{dA} \frac{A}{Q}$$

$$Q_d = -3P + 15 + 6A$$

Assume $P = \$10$, $A = \$5$, so $Q_d = -3 \cdot 10 + 15 + 6 \cdot 5 = 15$

Elasticities for Linear Demand Functions

- From a linear demand function, we can easily compute various elasticities.
- Given a linear demand function:

$$Q_x^d = \alpha_0 + \alpha_x P_x + \alpha_y P_y + \alpha_M M + \alpha_H H$$

- Own price elasticity: $E_{Q_x, P_x} = \alpha_x \frac{P_x}{Q_x}$.
- Cross price elasticity: $E_{Q_x, P_y} = \alpha_y \frac{P_y}{Q_x}$.
- Income elasticity: $E_{Q_x, M} = \alpha_M \frac{M}{Q_x}$.

Elasticities for Linear Demand Functions In Action

- The daily demand for Invigorated PED shoes is estimated to be:

$$Q_x^d = 100 - 3P_x + 4P_y - 0.01M + 2A_x$$

- Suppose good X sells at \$25 a pair, good Y sells at \$35, the company utilizes 50 units of advertising, and average consumer income is \$20,000. Calculate the own price, cross-price, and income elasticities of demand.
- $Q_x^d = 100 - 3(\$25) + 4(\$35) - 0.01(\$20,000) + 2(50) = 65$ units.
- Own price elasticity: $-3\left(\frac{25}{65}\right) = -1.15$.
- Cross-price elasticity: $4\left(\frac{35}{65}\right) = 2.15$.
- Income elasticity: $-0.01\left(\frac{20,000}{65}\right) = -3.08$.

A quick note on OLS and the Log function

- If you transform both sides of an OLS equation ($Y=a+bX$) by the log function ($\ln$), the result is an elasticity (percentage change).
- For example: $\ln(Y)=a + b\ln(X)$
- Here, the estimate for b is the elasticity of Y with respect to X .

Example:

- If you run a regression of a demand function and get

$$\ln Q_x = 4.81 - 0.85 \ln P_x$$

- This means that the own price elasticity of demand is -0.85

Elasticities for Nonlinear Demand Functions

- A log-linear specification is appropriate if quantity demanded is not linearly related to the explanatory variables:

$$\ln Q_x^d = \beta_0 + \beta_x \ln P_x + \beta_y \ln P_y + \beta_M \ln M + \beta_H \ln H$$

- Own price elasticity: β_x .
- Cross price elasticity: β_y .
- Income elasticity: β_M .

Elasticities for Nonlinear Demand Functions In Action

- An analyst for a major apparel company estimates that the demand for its raincoats is given by

$$\ln Q_x^d = 10 - 1.2 \ln P_x + 3 \ln R - 2 \ln A_y$$

- where R denotes the daily amount of rainfall and A_y the level of advertising on good Y .
- What would be the impact on demand of a 10 percent increase in the daily amount of rainfall?

$$E_{Q_x, R} = \beta_R = 3. \text{ So, } E_{Q_x, R} = \beta_R = \frac{\% \Delta Q_x^d}{\% \Delta R} \Rightarrow 3 = \frac{\% \Delta Q_x^d}{10}$$

- A 10 percent increase in rainfall will lead to a 30 percent increase in the demand for raincoats.

Multiple Regression to Estimate Demand Curves

- **Table 3-8**
- **Estimated Demand for Rental Units Using Multiple Regression**

	A	B	C	D	E	F	G
1	Observation	Quantity	Price	Advertising	Distance		
2	Average	53.10	420.00	20.50	5.70		
3							
4		<i>Coefficients</i>	<i>Standard Error</i>	<i>t-Statistic</i>	<i>P-Value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
5							
6	Intercept	135.15	20.65	6.54	0.0006	84.61	185.68
7	Price	-0.14	0.06	-2.41	0.0500	-0.29	0.00
8	Advertising	0.54	0.64	0.85	0.4296	-1.02	2.09
9	Distance	-5.78	1.26	-4.61	0.0037	-8.86	-2.71

- Functional relationships with multiple variables:
- For linear demand relationship.

$$Q_x^d = \alpha_0 + \alpha_x P_x + \alpha_y P_y + \alpha_M M + \alpha_H H + e$$

- Or for non-linear relationship.

$$\ln Q_x^d = \beta_0 + \beta_x \ln P_x + \beta_y \ln P_y + \beta_M \ln M + \beta_H \ln H + e$$